Perturbation Theory and Mixture Models: Application to Particle Physics

ACAS
21 Oct 2004

Cyrus Taylor
Dept. of Physics
Case Western Reserve University
cct@case.edu

(work joint with C. Loader and R. Pilla)

Outline

- Review of score + formula for asymptotic distribution – do we need additional parameters to describe the data?
- Applications to particle physics
  - Ex: search for new particle resonances
  - Ex: energy spectrum of highest energy cosmic rays

Mixture models, score statistic and its asymptotic distribution

- Mixture models:

$$p(x;\tilde{n},\grave{o}) = (1 \grave{a} \tilde{n})f(x) + \tilde{n}\ (x;\grave{o})$$

- Score statistic

$$S^{?}(x;\grave{o}) := p\frac{S(x;\grave{o})}{n\,C(\grave{o};\grave{o}^{0})}$$

$$= p\frac{1}{n\,C(\grave{o};\grave{o}^{0})}\ '\ \sum_{i=1}^{n}\frac{(x_i;\grave{o})}{f(x_i)} \grave{a}\ 1$$

- where $C(\grave{o};\grave{o}^{0}) = \int_{f}\frac{(x;\grave{o})g^{2}}{f(x)}dx \grave{a}\ 1$

Asymptotic distribution

$$\Pr\left[\sup_{t \in T} Z(t) > c\right] \approx \frac{V_0}{A_{d+1}} \Pr(\chi^2_{d+1} > c^2) + \frac{L_0}{2A_d} \Pr(\chi^2_d > c^2)$$

- V_0 described by d-dimensional volume of manifold expressible through covariance function

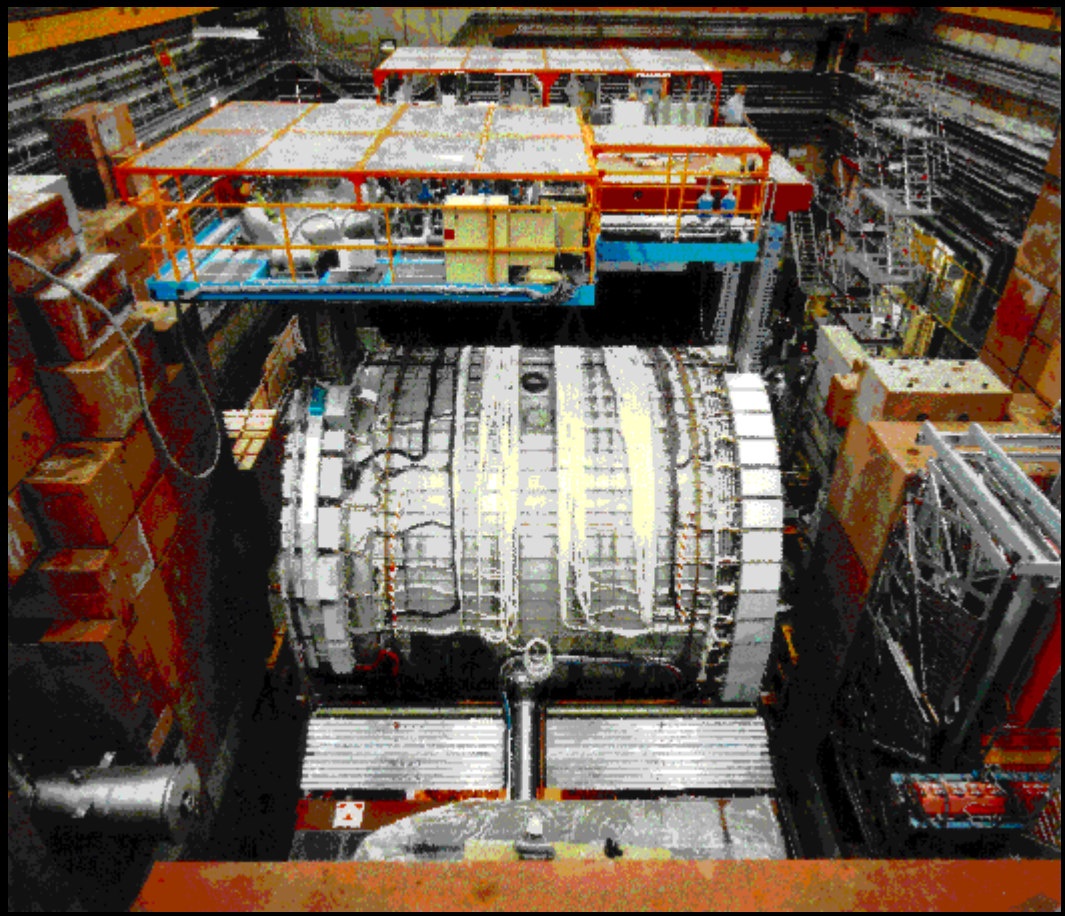
Ex: search for new resonances

- What are physicists searching for?
- Why are we searching for it?
- How do we search for it?

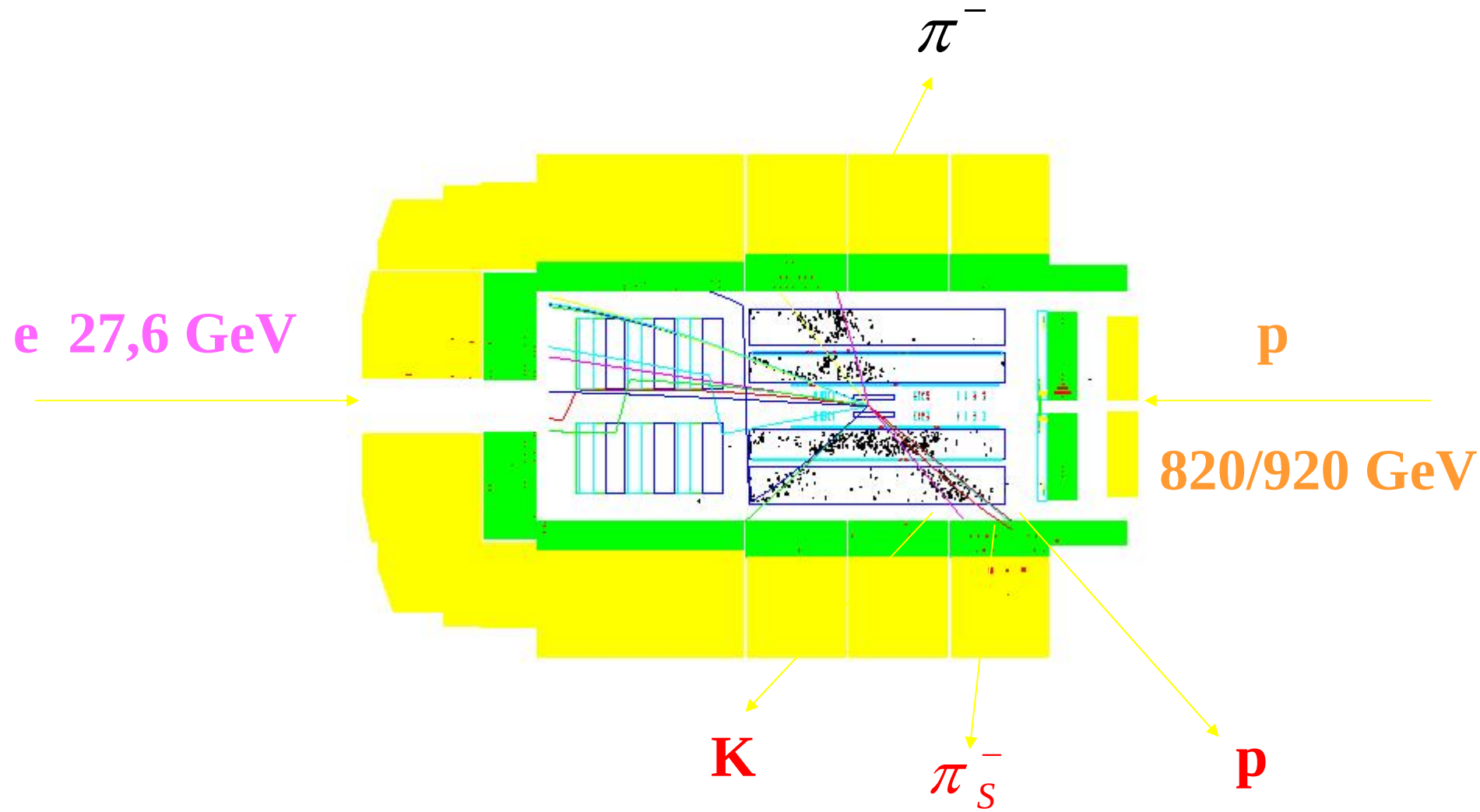
Ex: Pentaquarks

- QCD – nobel prize
- Q q-bar, 3 q states
- Pentaquark discovered
- Charmed pentaquarks
- H1 claims discovery; Zeus doesn’t see it

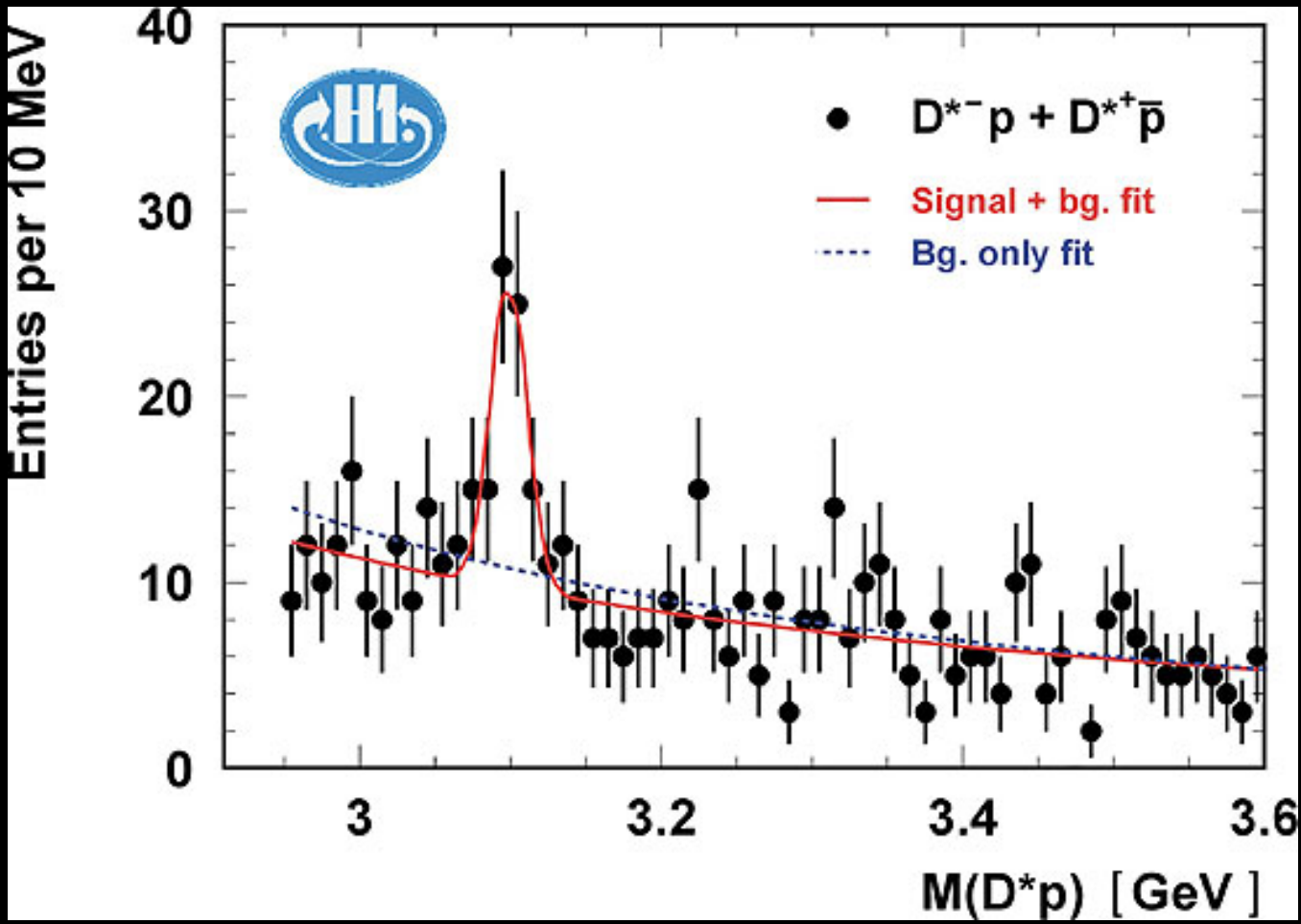
H1 detector

- 

Pentaquark in H1 setup



Ex: pentaquarks



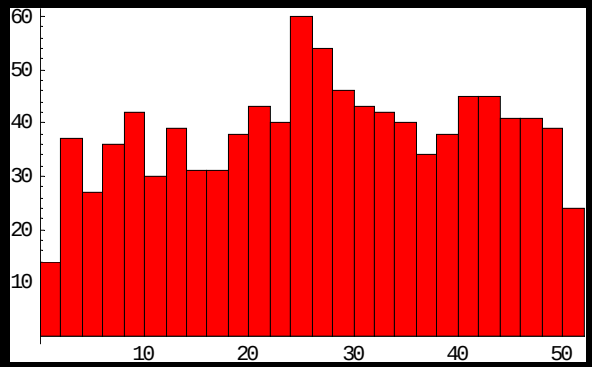
Mixture models in particle physics

- Background : power law
- Perturbation (resonance): Breit-Wigner (Cauchy):

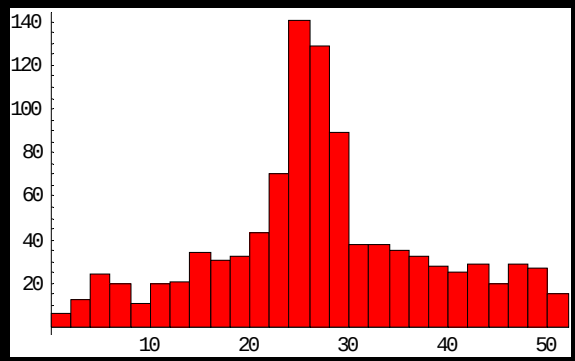
$$(E; E_0) = \Gamma/(2\pi)\,[(E - E_0)^2 + (\Gamma/2)^2]^{-1}$$

Application of score analysis (MC)

- 10% mixture

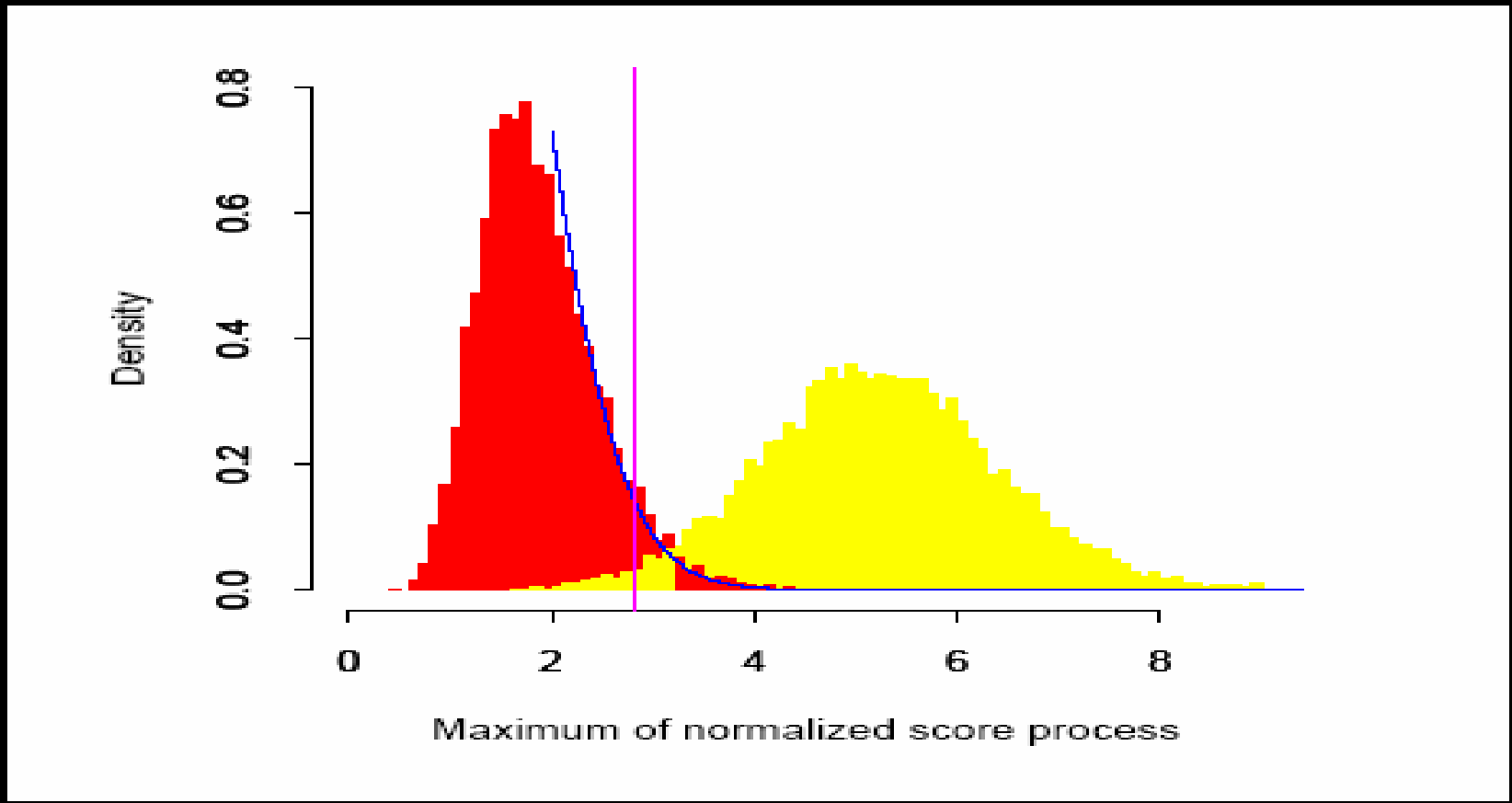


- 50% mixture

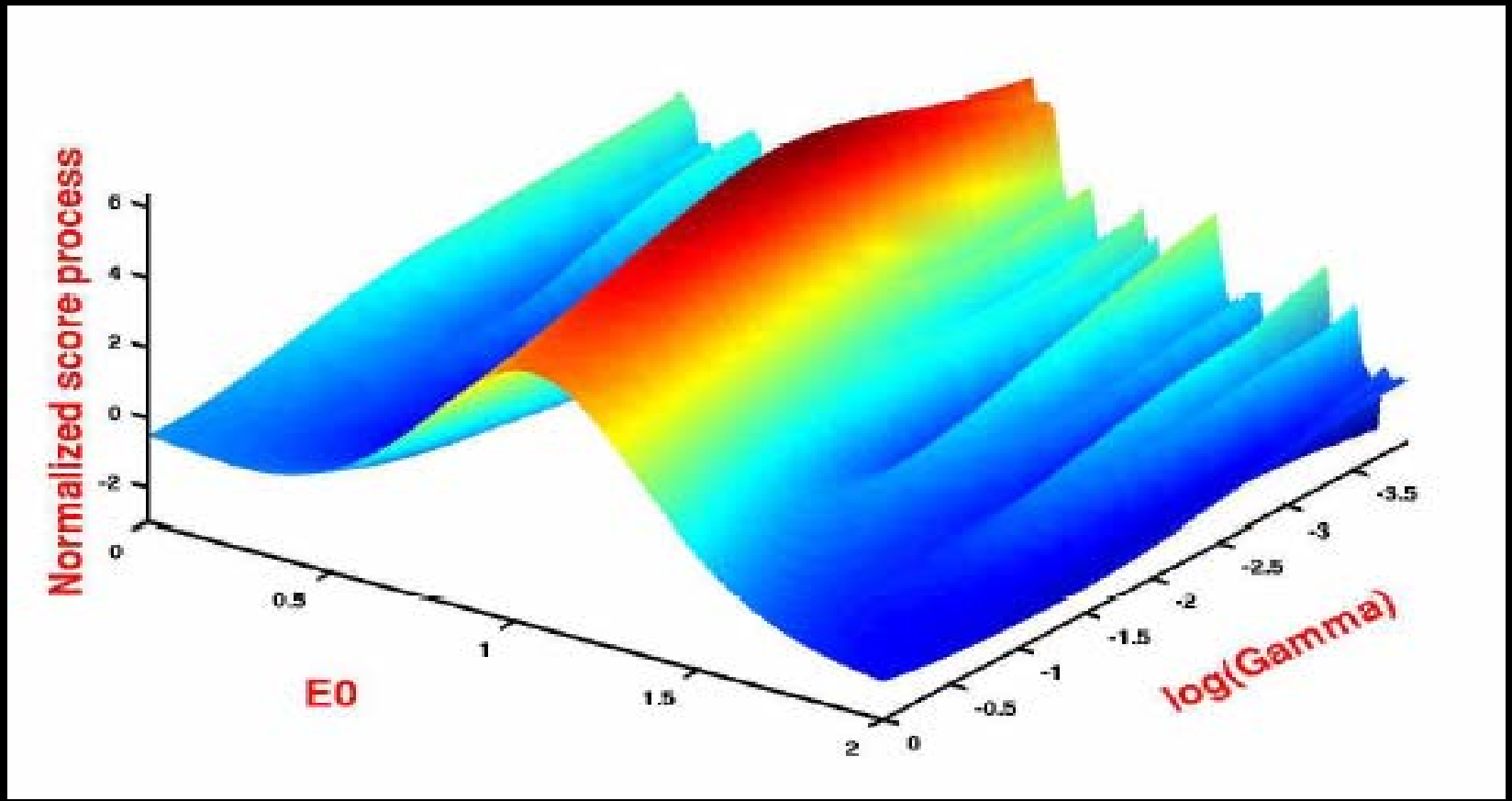


- Model parameters:

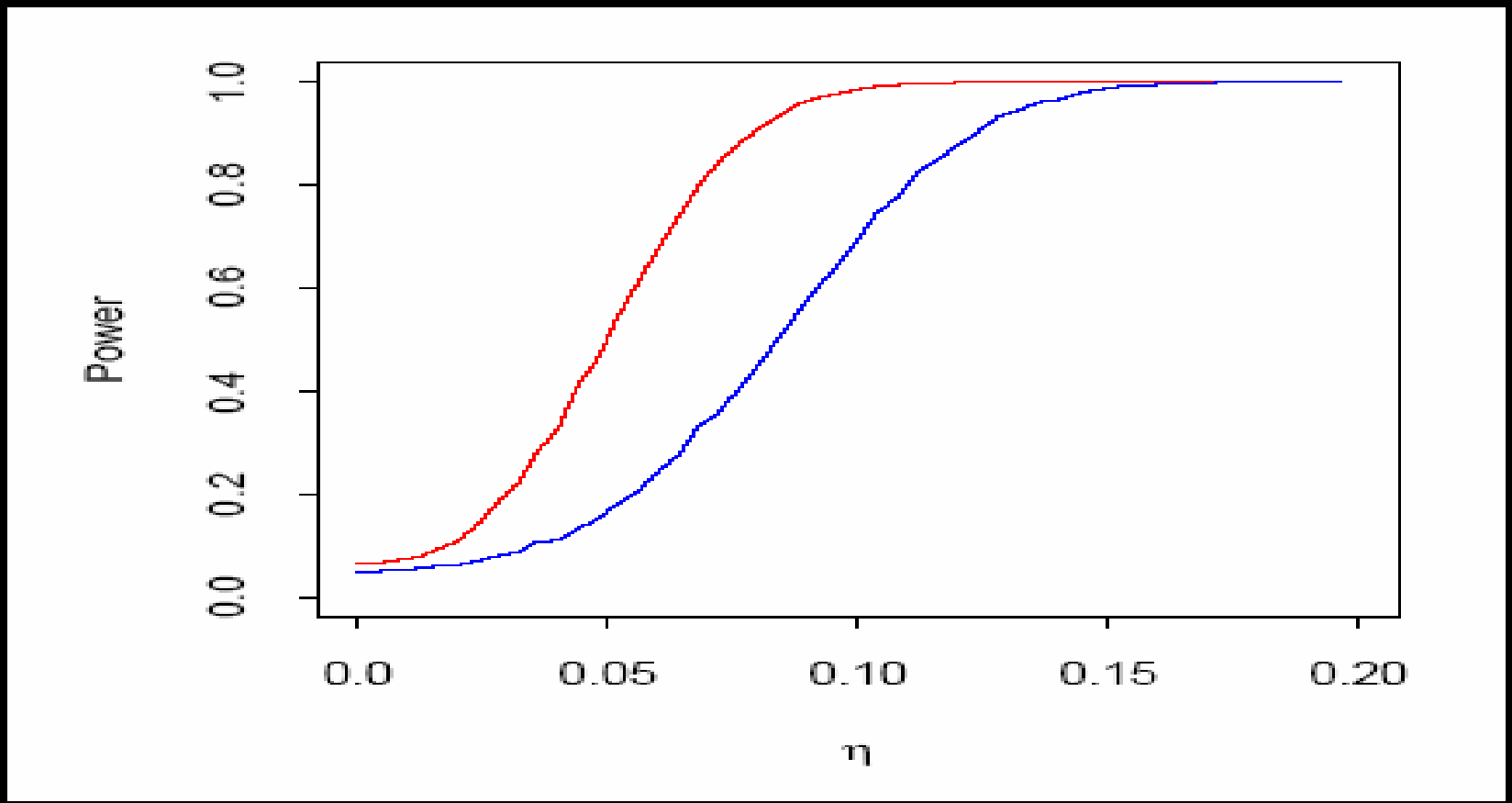
T: ñ = 0:1


Density
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
Maximum of normalized score process

Surface of normalized score process



Power of $\ddot{y}^2$ and normalized score

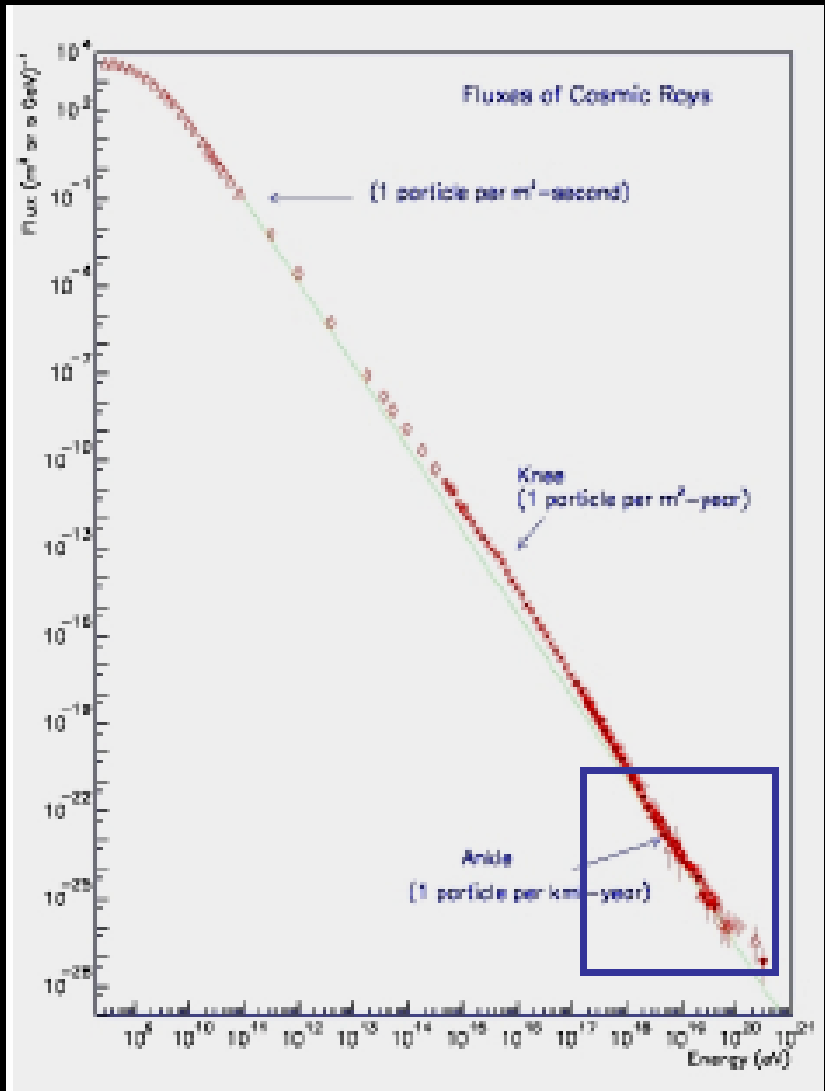


Next: LHC

- Search for Higgs
- Search for SUSY
- Search for the unexpected

Cosmic Rays

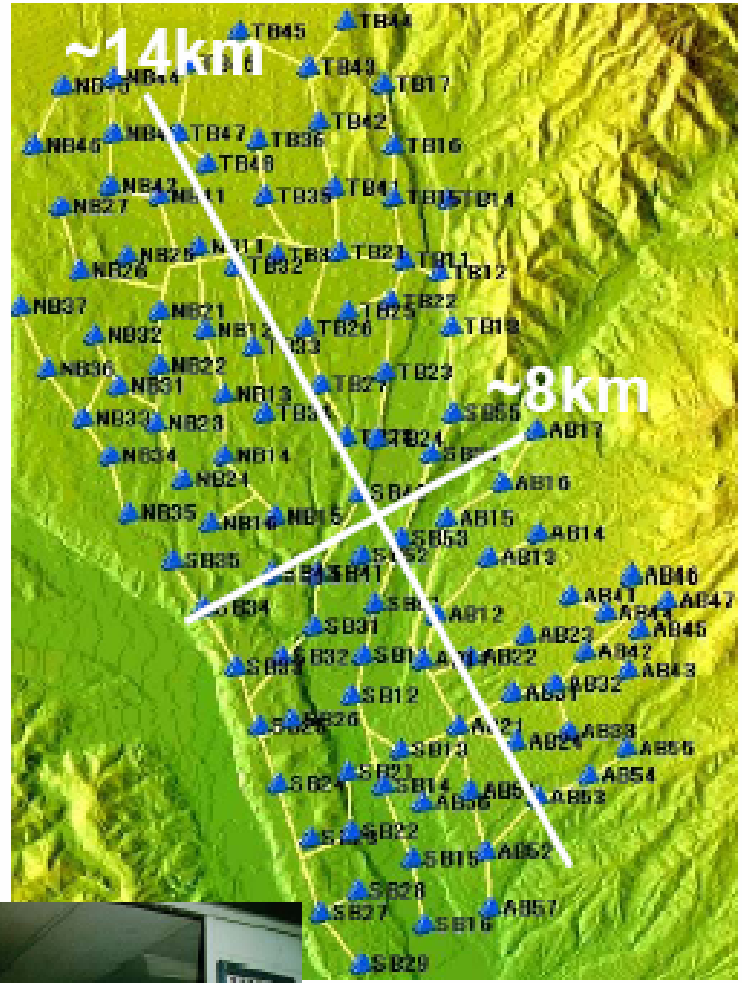
- spectrum



AGASA

Akeno Giant Air Shower Array

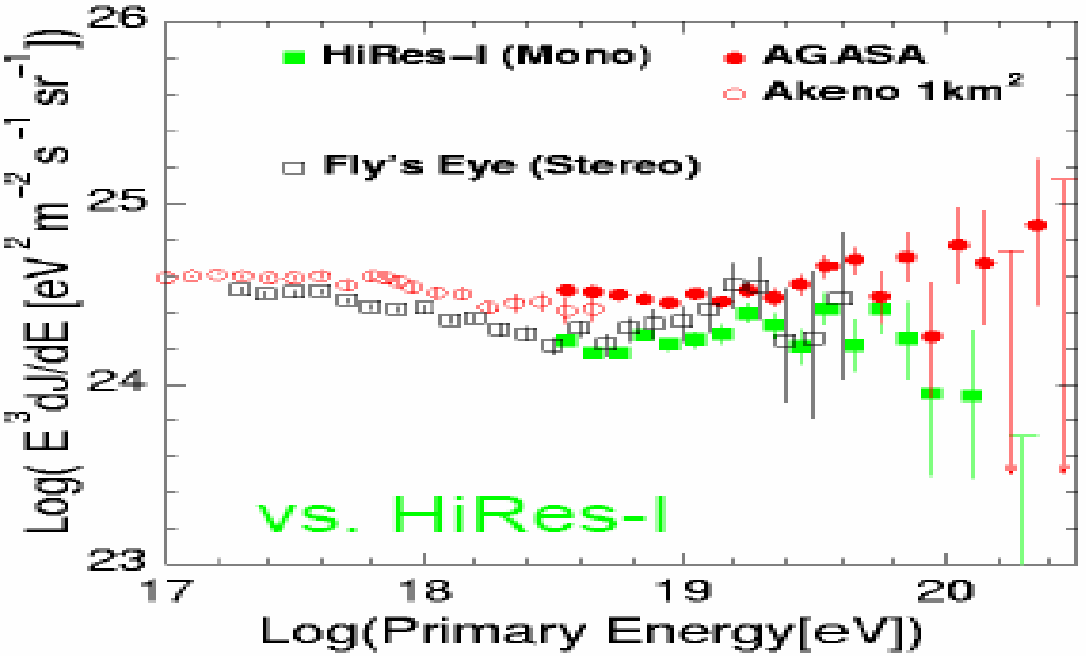
- **Detectors**
  - **111 surface detectors (2.2m²)**
    - 5cm thinck scintillator
    - Optical fibre cable to observatory
      - Delay time monitored @100ps accuracy
    - Location suveyed: $\Delta x,y$=0.1m; Δz=0.3m
  - **27 muon detectors (2.8–10m²)**
    - Fe / concrete absorber
      +proportional counters
    - E_{th}>0.5GeV



- **Operation**
  - Started in February 1990
    up to now ~95% live ratio
  - **We will shut down**
    at the end of this year…;_;

High End of spectrum

Recent spectra
(AGASA vs. HiRes@Tsukuba ICRC)



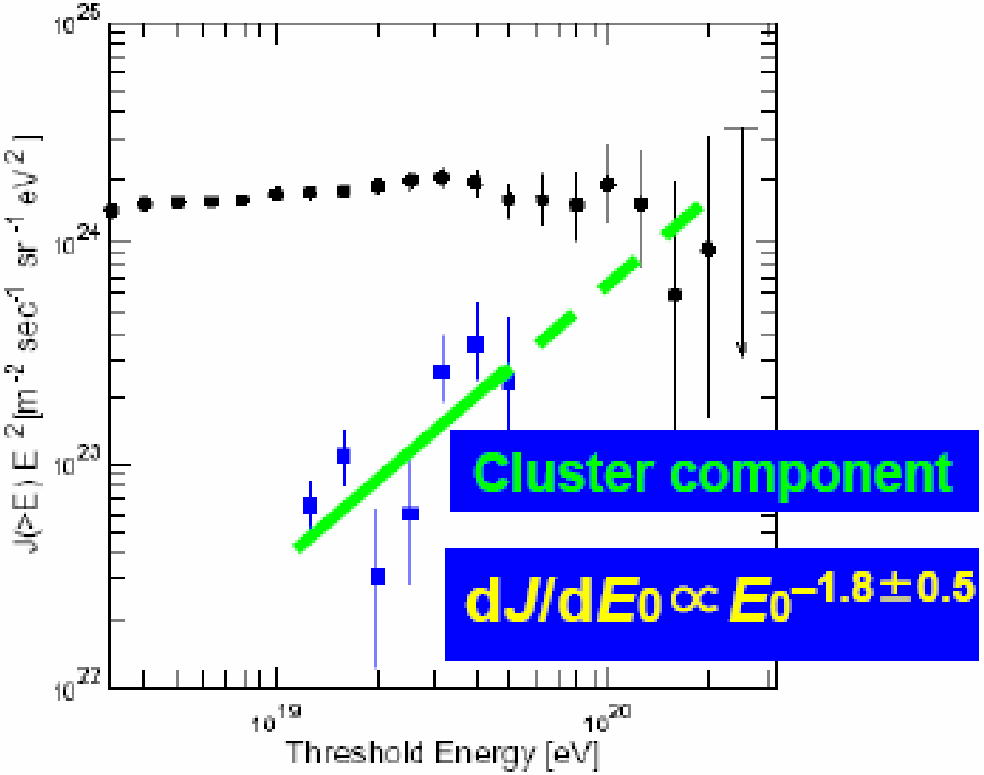
- ~2.5 sigma discrepancy between AGASA & HiRes

Greissen cutoff

- Shouldn’t be any very high energy cosmic rays – interactions with microwave background radiation
- Where are they coming from? New sources?
- Auger project: $50 million air shower array

Auger…New component?

- **Integral EHECR spectrum**
  (Ordinary EHECR vs. cluster comp.)



Score sensitivity for power-law mix

$$p_\gamma(x) = \frac{1}{B_\gamma} x^{-\gamma}$$

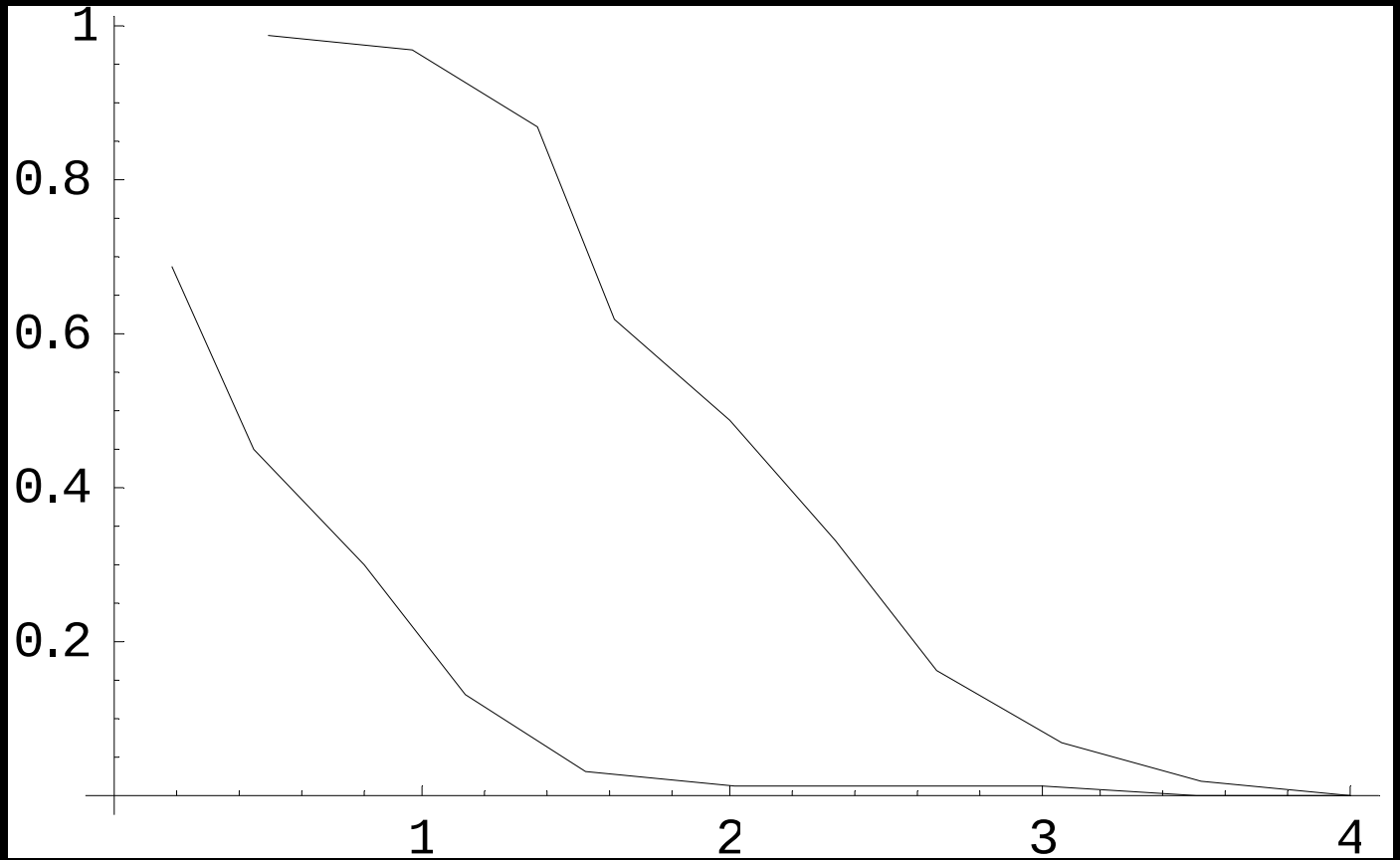
$$B_\gamma = \int_a^b x^{-\gamma}\, dx$$

$$H_0 = p_\varepsilon(x)$$

$$H_1 = (1-\sigma)\, p_\varepsilon(x) + \sigma\, p_\mu(x); \quad \mu < \varepsilon$$

- Ex: slightly softer (~2.7 added in at 20% level); cut off at region where disagreement begins

Sensitivity $\tilde{n} = 0.2; N = 1000$



Conclusions

- Score test statistic + asymptotic distribution represents a powerful new tool for the search for new physics in high energy physics and particle astrophysics