# Modeling of Tank Gun Accuracy Under Two Different Zeroing Methods

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The accuracy of weapon systems firing unguided ammunition, such as most tank and artillery systems, relies on understanding numerous sources of error and correcting for them. Random errors are addressed through weapon and ammunition design, quality production, effective maintenance, and firing processes that minimize the magnitude of these error sources. Bias errors are different because their magnitude and direction can often be estimated and then compensated for while aiming the weapon. In this paper, we examine two common methods used to compensate for the bias errors inherent in aiming and firing tank cannons. There are a number of advantages and disadvantages to each of these processes, but the relative accuracy benefits depend on the magnitude of the error sources that make up the total error budget for the fleet of tanks. Although this budget is comprised of numerous error sources, reasonable estimates of accuracy are attainable with basic models of the predominant factors. This presentation illustrates models for first-shot accuracy under two common zeroing processes. Such a comparison can then be used to determine the policy that makes a particular fleet of tanks most accurate in varying tactical situations and can be a useful tool for identifying and reducing the actual source of bias errors.

In tanks, estimating the bias error is a two-part process. First, boresighting is a process that measures the offset between the tank's fire control system and its cannon, and then corrects for that offset. Typically, boresighting is accomplished by placing an optical device along the centerline of a cannon. The cannon is then moved so that a target point, at a known distance from the tank, is viewed with the aid of the optical device. Assuming that there is no error in the optics of the device and that the device can be placed directly along the cannon centerline, the cannon will be aimed at the target point. Of course there are errors, but modern boresighting devices are relatively accurate.[1] Once the cannon is aimed at the target point, the tank's fire control optics are also aimed at the target point. Since the tank-to-target distance is known and the various angles of the cannon and fire control optics can be measured, the boresighting process provides enough geometrical information to aim the tank's cannon at a target, at any range, with the tank's fire control optics.

While boresighting allows accurate alignment of the cannon with respect to the target prior to firing the cannon, the cannon can bend, displace laterally, and rotate during the actual firing process. The result is that by the time the projectile exits the cannon muzzle, the cannon is no longer aimed so precisely at the target. Additional mechanical interactions and aerodynamics further alter the eventual trajectory of the projectile. The sum of the changes between the predicted trajectory (as based on the statically aimed cannon) and the actual trajectory is often referred to as "jump". Correcting for jump, which normally has a large, nonrandom component, is called zeroing. Zeroing is the second step in the two-part process of estimating and compensating for bias error. It can be accomplished a number of ways; but the two most common in tank gunnery are individual zero and fleet zero. Tanks are individually zeroed when their jump is individually estimated, and a unique correction is calculated and applied to that tank. Fleet zero is a process by which an average jump value is estimated for a number (fleet) of tanks and the correction implied by that average jump is applied to every tank in the fleet.

Currently, when tanks are individually zeroed, it is done manually. In other words, the tanks are taken to a firing range, where they shoot a number of rounds at a target, the impact points are measured relative to the aim point, and a correction is calculated and applied based on the average miss distance and direction. Since each ammunition type reacts differently, the tank must be individually zeroed with each ammunition type it will fire. This method of zeroing was used in the United States into the 1980s.

[1] Held, B. J., and D. W. Webb. "A Comparison of Muzzle Boresights for Tank Cannon." BRL-MR-3977, U.S. Army Ballistics Research Laboratory, Aberdeen Proving Ground, MD, 1992. Prior to using dedicated boresight devices, one method of boresighting in the United States was to tape string across the muzzle of cannon to form a crosshair. Binoculars were then used to view down the cannon from the breech end, across the crosshair at the muzzle, to the target point.

However, with the advent of depleted uranium ammunition and the 120mm cannon, new calibration policies had to be considered. The new ammunition was considered environmentally unfriendly, hence there was a desire to limit its use. Additionally, the new, higher technology and often larger caliber ammunition being introduced was expensive, so finding a way to zero without firing a lot of rounds was important. This requirement led to the fleet zero technique of estimating and compensating for bias errors.

The major assumption behind the fleet zero concept is that all the tanks in a fleet shoot in a similar manner; for example, if Tank X shoots a particular ammunition type high and to the right, the assumption requires that Tank Y would shoot the same ammunition type high and to the right. If this basic assumption is reasonably accurate, then a good estimation of the bias error for Tank X will also be a good bias estimation for Tank Y. Current fleet zero polices depend on this assumption. Today, the U.S. Army has established a fleet zero computer correction factor (CCF)[*] for each fielded ammunition type. The fleet zero facilitates the introduction of new ammunition types since part of the fielding process is to establish and distribute a CCF for the new round. This is accomplished by firing the ammunition, under several firing conditions, from a number of tanks representative of the larger fleet and measuring the miss distances to the aim point. The basic data is then analyzed and generalized to a single CCF for the entire fleet for the new ammunition type. In the case of the individual zero, introduction of a new ammunition type requires each tank in the fleet to fire several of the rounds to establish CCFs unique to each tank.

Using the fleet zero has a number of other distinct advantages over an individual zero policy. The limited number of rounds required to establish a fleet CCF substantially reduces the cost compared to individually zeroing tanks. The controlled nature of the tests used to establish the fleet CCF also indicates that safety and environmental concerns associated with the firing of tactical rounds can be greatly diminished. As individually zeroing tanks requires safe firing ranges near the battlefield and the need to supply additional ammunition for zeroing, the fleet zero policy is more tactically sound since it eliminates these requirements. Finally, one of the most significant advantages fleet zero offers is the ease with which it is implemented at the tank and soldier level. Once a CCF number is passed to a tank owning unit all that is required is that the CCF be entered tank fire control computers. In contrast, individually zeroing tanks requires firing the tanks, measuring the impacts, calculating the CCF, and then entering it on the fire control computer. Each of these steps, which can produce significant error, requires training.

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[*] The CCF is the value of the correction that is input into the fire control computer of the tank. The value is used by the fire control computer as part of the calculation that aims the cannon.

There are disadvantages to the fleet zero policy, however. The greatest disadvantage is that the fleet zero policy is only as good as the basic assumption that underlies it; hence, fleet zero does not account for differences between tanks and firing conditions. When that assumption is found wanting, tanks that shoot significantly different than the fleet average will shoot inaccurately under the fleet zero policy. Additionally, if the conditions under which a tank fires (e.g., the ammunition temperature) has a significant affect on the magnitude and direction of the bias error, a fleet zero policy may not adequately address the affect. Individually zeroing tanks obviously accounts for the firing differences between them, and if done under conditions similar to those encountered during battle or gunnery drills, will also account for ammunition bias sensitivity to environmental conditions.

Choosing the better zeroing policy for a given set of conditions requires an understanding of how those conditions affect accuracy as they vary. Since it is impractical to obtain this experimentally, a mathematical model is necessary.

**Mathematical Modeling of Fleet Zero**

For our modeling purposes, the three dominant components of variance associated with tank cannon accuracy are round-to-round variance, occasion-to-occasion variance, and tank-to-tank variance. These components are quantified by the parameter values $\sigma_R^2$, $\sigma_O^2$, and $\sigma_T^2$, respectively. Round-to-round errors are random differences in jump between the same type of rounds fired on the same occasion from the same tank. An occasion is defined as a total firing event during which nothing has happened to the tank that could appreciably affect how it shoots (e.g., maintenance or significant movement). In other words, the total occasion timeframe is short enough that neither the tank nor firing conditions significantly changing is assured. Occasion-to-occasion errors are therefore the random differences between firing occasions (in regard to their mean jumps) while firing the same type of ammunition from the same tank. Finally, tank-to-tank errors are the differences in mean jumps between different tanks firing the same kind of ammunition. Tank-to-tank error is only pertinent to the fleet zero method of zeroing. In addition to the three major errors, ammunition temperature is included in the model as the major known variable not otherwise accounted for in the fire control calculations of current U.S. tanks.

The three variance components and ammunition temperature are the only contributors to the total variability in the Stationary-to-Stationary Ammunition Accuracy Test (SSAAT), which is the U.S. Army

Test Center's protocol for initially estimating the CCF. As displayed in Table 1, the SSAAT calls for four randomly chosen tanks to each fire four three-round occasions. Within tanks, each occasion is fired at one of four specified ammunition temperatures. The CCF is then calculated as a weighted average of the mean impacts from the firings at the four temperature conditions. The weightings are based on the U.S. Army Materiel and Systems Analysis Activity's (AMSAA) estimated global temperature distribution for armored vehicle combat.

| Tank | Temperature ($^{o}$F) | | | |
|---|---|---|---|---|
| | -10 | 30 | 70 | 110 |
| 1 | Occasion 1: Rounds 1-3 | Occasion 5: Rounds 13-15 | Occasion 9: Rounds 25-27 | Occasion 13: Rounds 37-39 |
| 2 | Occasion 2: Rounds 4-6 | Occasion 6: Rounds 16-18 | Occasion 10: Rounds 28-30 | Occasion 14: Rounds 40-42 |
| 3 | Occasion 3: Rounds 7-9 | Occasion 7: Rounds 19-21 | Occasion 11: Rounds 31-33 | Occasion 15: Rounds 43-45 |
| 4 | Occasion 4: Rounds 10-12 | Occasion 8: Rounds 22-24 | Occasion 12: Rounds 34-36 | Occasion 16: Rounds 46-48 |

Table 1. Design layout for the Stationary-to-Stationary Ammunition Accuracy Test (SSAAT).

Under more general conditions, a linear model for the SSAAT is given by

$$y_{ijkl} = \alpha + T_i + \beta\tau_j + O_{k(ij)} + R_{l(ijk)},$$

where

(1) $y_{ijkl}$ is the azimuth (or elevation) jump for round $l$ fired during occasion $k$ from tank $i$ at temperature $j$, for $i$ = 1, …, $a$; $j$ = 1, …, $b$; $k$ = 1, …, $c$; and $l$ = 1, …, $n$;

(2) $\alpha$ is the intercept from the temperature dependency model;

(3) $T_i$ is the effect of tank $i$, assumed to be $N\left(0, \sigma_T^2\right)$;

(4) $\beta$ is the slope from the temperature dependency model;

(5) $\tau_j$ is the $j^{\text{th}}$ ammunition temperature level;

(6) $O_{k(ij)}$ is the effect of occasion *k*, nested within the combination of tank *i* and temperature *j*, assumed to be $N\left(0,\sigma_O^2\right)$; and

(7) $R_{l(ijk)}$ is the effect of round *l* from occasion *k* fired from tank *i* at temperature *j*, assumed to be $N\left(0,\sigma_R^2\right)$.

A few assumptions concerning this model deserve further discussion. First, although impact data are bivariate, the model is univariate as a result of the assumption of independence between horizontal (azimuth) and vertical (elevation) impacts. This assumption is invalid for some weapon systems, especially rapid-cadence systems; however, for large-caliber weapons in which the cadence is relatively slow, this assumption has proven consistent over many years of testing. Second, the temperature dependency model previously noted is a simple linear regression model relating jump as a function of ammunition temperature. This relationship between ammunition temperature and jump is the result of (1) the relationship between propellant temperature and in-bore velocity (hot propellant burns quicker, hence the projectile travels faster while in-bore) and (2) the dynamic and consistent motion of the gun tube immediately after shot initiation. The coefficients from this model are estimated from previous live-fire tests of this ammunition type. Over the range of ammunition temperatures in the SSAAT, a simple linear regression has shown to be a basic, yet adequate model.[2] Finally, based upon the belief that the effect of ammunition temperature is the same upon each tank, the model excludes an interaction term involving these two variables.

In the SSAAT model, the average jump at each temperature is given as $\bar{y}_{\tau_j} = \alpha + \bar{T}_\bullet + \beta\tau_j + \bar{O}_{\bullet(\bullet j)} + \bar{R}_{\bullet(\bullet j \bullet)}$, and under the random-effects assumptions, is normally distributed with mean $\alpha + \beta\tau_j$ and variance $\frac{1}{a}\sigma_T^2 + \frac{1}{ac}\sigma_O^2 + \frac{1}{acn}\sigma_R^2$. For a specific set of weights, $\{w_1,\ldots,w_b\}$ satisfying $\sum_{j=1}^{b} w_j = 1$, the CCF is given by $CCF = \sum_{j=1}^{b} w_j \bar{y}_{\tau_j} =$ $\alpha + \beta\sum_{j=1}^{b} w_j\tau_j + \sum_{j=1}^{b} w_j\bar{T}_\bullet + \sum_{j=1}^{b} w_j\bar{O}_{\bullet(\bullet j)} + \sum_{j=1}^{b} w_j\bar{R}_{\bullet(\bullet j \bullet)}$. Therefore, the CCF is also a normally distributed random variable having mean $\alpha + \beta\sum_{j=1}^{b} w_j\tau_j$ and variance $\frac{\sigma_T^2}{a} + \frac{\sigma_O^2}{ac}\sum_{j=1}^{b} w_i^2 + \frac{\sigma_R^2}{acn}\sum_{j=1}^{b} w_i^2$.

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[2] Held, B. J., D. W. Webb and E. M. Schmidt, "Temperature Dependent Jump of the 120mm M256 Tank Cannon." BRL-MR-3927, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 1991.

After the SSAAT is completed, each tank in the fleet incorporates the CCF into its fire control system. Now, tanks in the fleet firing this same ammunition at perhaps a different temperature, $\tau_{j'}$, will have a first-shot jump which can be similarly modeled as $y_{i'j'k'l'} = \alpha + T_{i'} + \beta\tau_{j'} + O_{k'(i'j')} + R_{l'(i'j'k')} - CCF$. This jump is also normally distributed with mean $\beta\left(\tau_{j'} - \sum_{j=1}^{b} w_j \tau_j\right)$ and variance $\left(\frac{a+1}{a}\right)\sigma_T^2 + \left(1 + \frac{1}{ac}\sum_{j=1}^{b} w_j^2\right)\sigma_O^2 + \left(1 + \frac{1}{acn}\sum_{j=1}^{b} w_j^2\right)\sigma_R^2$. This is a general model which can be applied to either the horizontal and vertical jumps. However, note that for horizontal data, it is assumed that $\beta = 0$, since the temperature dependency on jump is assumed to exist only in the vertical direction.*

**Mathematical Modeling of Individual Zero**

The mathematical model for the SSAAT provides a framework for a model of those rounds fired during an individual zeroing exercise. Despite the fact that only one occasion (at one temperature) is fired per tank, all factors from the SSAAT model are retained. Because $b = c = 1$, the subscripts for temperature and occasion are no longer necessary and, consequently, can be dropped. The subscript $i$ is retained as an identifier for tanks and to accentuate the fact that the individual zero is different for every tank. Therefore, rounds fired during the exercise are modeled as $y_{il} = \alpha + T_i + \beta\tau + O + R_{l(i)}$. To avoid confusion with the SSAAT model, we let $l$ range from 1 to $m$. The individual zero for tank $i$ will simply be the average jump observed during its zeroing exercise, $IZ_i = \bar{y}_{i\bullet} = \alpha + T_i + \beta\tau + O + \bar{R}_{\bullet(i)}$, where $IZ_i$ is distributed normally with mean $\alpha + \beta\tau$ and variance $\sigma_T^2 + \sigma_O^2 + \frac{\sigma_R^2}{m}$.

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* Mass imbalances in the tank gun system cause a torque about its center of mass, resulting in movement of the gun tube while the projectile is in bore. Since the projectile's acceleration and velocity vary with the propellant temperature, the projectile's in-bore time also varies with the temperature. Because of the torque-induced gun tube motion, the muzzle-pointing direction at shot exit also varies with propellant temperature, resulting in temperature-related gun jump. The gun system's mass imbalance is primarily in the vertical plane, and the gun tube is more constrained in the horizontal plane due to the trunnions; thus, most of the described gun tube motion and resulting temperature-related gun jump is in the vertical plane.

That same tank firing at a later time and possibly at a different temperature, $\tau'$, will have its first-shot vertical jump modeled as $y_{il'} = \alpha + T_i + \beta\tau' + O' + R_{l'(i)} - IZ_i$, where $y_{il'}$ is distributed normally with mean $\beta(\tau' - \tau)$ and variance $2\sigma_{O_y}^2 + \frac{m+1}{m}\sigma_{R_y}^2$. Already we see that it is important for the ammunition temperature at the time of zeroing to be as close to the ammunition temperature during combat so that the jump bias from the intended aimpoint is minimal. For horizontal jumps, we follow a similar argument, except that there is no temperature dependency (i.e., $\beta = 0$), and conclude that $x_{ij'k'l'} \sim N\left(0, 2\sigma_{O_x}^2 + \frac{m+1}{m}\sigma_{R_x}^2\right)$.

## Hit Probability Comparison of the Two Zeroing Methods

Given the distributions of first-shot jumps under the two zeroing methods, determining which method offers the higher hit probability is of interest. To do this, we will assume that tanks are properly boresighted and aimed at the center of a square target of length $L$ (mils).

Under a fleet zero policy, first-shot hit probability is given in general by

$$P\left(\frac{-L}{2} < X_{FZ} < \frac{L}{2}\right) \times P\left(\frac{-L}{2} < Y_{FZ} < \frac{L}{2}\right)$$

$$= \left\{2\Phi\left(\frac{L}{2\sigma_{FZ_x}}\right) - 1\right\} \times \left\{\Phi\left(\frac{\frac{L}{2} - \beta\left(\tau_{i'} - \sum_{i=1}^{a} w_i \tau_i\right)}{\sigma_{FZ_y}}\right) - \Phi\left(\frac{\frac{-L}{2} - \beta\left(\tau_{i'} - \sum_{i=1}^{a} w_i \tau_i\right)}{\sigma_{FZ_y}}\right)\right\},$$

where $\Phi(u) = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$ (the cumulative normal distribution function), $\sigma_{FZ_x} = \sqrt{\left(\frac{a+1}{a}\right)\sigma_{T_x}^2 + \left(1 + \frac{1}{b}\sum_{i=1}^{a} w_i^2\right)\sigma_{O_x}^2 + \left(1 + \frac{1}{bn}\sum_{i=1}^{a} w_i^2\right)\sigma_{R_x}^2}$, and $\sigma_{FZ_y}$ is defined similarly.

Recall from Table 1 that the current SSAAT calls for $a = 4$, $b = 4$, $c = 1$, and $n = 3$; additionally, the ammunition temperatures (measured in degrees Fahrenheit) are $\tau_1 = -10$, $\tau_2 = 30$, $\tau_3 = 70$ and $\tau_4 = 110$. The weights are $w_1 = .01$, $w_2 = .19$, $w_3 = .64$, and $w_4 = .16$, representing AMSAA's estimated global temperature distribution for armored vehicle combat (i.e., AMSAA expects

1% of future tank battles to take place in a –10$^{o}$ F environment, 19% of future tank battles to take place in a 30$^{o}$ F environment, and so on). With these substitutions, we obtain $x_{i'j'k'l'} \sim N\left(0, 1.25\sigma_{T_x}^2 + 1.12\sigma_{O_x}^2 + 1.04\sigma_{R_x}^2\right)$ in the horizontal direction; for vertical jumps, $y_{i'j'k'l'} \sim N\left(\beta\left(\tau_{j'} - 68\right), 1.25\sigma_{T_y}^2 + 1.12\sigma_{O_y}^2 + 1.04\sigma_{R_y}^2\right)$. Therefore, the first-shot hit probability under a fleet zero policy is given by

$$P(Hit_{FZ}) = \left\{2\Phi\left(\frac{L}{2\sigma_{FZ_x}}\right) - 1\right\} \times \left\{\Phi\left(\frac{\frac{L}{2} - \beta\left(\tau_{i'} - 68^o F\right)}{\sigma_{FZ_y}}\right) - \Phi\left(\frac{\frac{-L}{2} - \beta\left(\tau_{i'} - 68^o F\right)}{\sigma_{FZ_y}}\right)\right\},$$

where $\sigma_{FZ_x} = \sqrt{1.25\,\sigma_{T_x}^2 + 1.12\,\sigma_{O_x}^2 + 1.04\,\sigma_{R_x}^2}$ and $\sigma_{FZ_y}$ is similarly defined. For some types of ammunition, the elevation jump and propellant temperature are independent. In this case, we have $\beta = 0$ so that $P(Hit_{FZ}) = \left\{2\Phi\left(\frac{L}{2\sigma_{FZ_x}}\right) - 1\right\} \times \left\{2\Phi\left(\frac{L}{2\sigma_{FZ_y}}\right) - 1\right\}$.

For individual zero, the hit probability is given by

$$P\left(\frac{-L}{2} < X_{IZ} < \frac{L}{2}\right) P\left(\frac{-L}{2} < Y_{IZ} < \frac{L}{2}\right) = \left\{2\Phi\left(\frac{L}{2\sigma_{IZ_x}}\right) - 1\right\}\left\{\Phi\left(\frac{\frac{L}{2} - \beta\left(\tau_{i'} - \tau_i\right)}{\sigma_{IZ_y}}\right) - \Phi\left(\frac{\frac{-L}{2} - \beta\left(\tau_{i'} - \tau_i\right)}{\sigma_{IZ_y}}\right)\right\},$$

where $\sigma_{IZ_x} = \sqrt{2\sigma_{O_x}^2 + \frac{m+1}{m}\sigma_{R_x}^2}$ and $\sigma_{IZ_y}$ is similarly defined. Most proponents of individual zeroing policy argue that as few as three rounds give an adequate estimate of a tank's zero and keep the cost of individually zeroing the whole fleet to a minimum. Therefore, setting $m = 3$, $\sigma_{IZ_x} = \sqrt{2\sigma_{O_x}^2 + 1.33\,\sigma_{R_x}^2}$. Finally, in the specific case of ammunition for which $\beta = 0$, we obtain an expression analogous to that for fleet zero, $P(Hit_{IZ}) = \left\{2\Phi\left(\frac{L}{2\sigma_{IZ_x}}\right) - 1\right\} \times \left\{2\Phi\left(\frac{L}{2\sigma_{IZ_y}}\right) - 1\right\}$.

Under specific conditions defined by the parameter values, one can determine which zeroing method has a higher hit probability. For example, under the current SSAAT procedure and an individual zeroing exercise with $m = 3$, if one assumes (1) no temperature dependency (i.e., $\beta = 0$), (2) equivalence of horizontal and vertical variances (i.e., $\sigma_{T_x}^2 = \sigma_{T_y}^2$, $\sigma_{O_x}^2 = \sigma_{O_y}^2$, and $\sigma_{R_x}^2 = \sigma_{R_y}^2$), and (3) that the occasion-to-occasion and round-to-round variances are equal, then $P(Hit_{FZ}) > P(Hit_{IZ})$ if and only if $\sigma_T < .97\sigma_R$. In most cases, however, such a simple relationship cannot be established.

Given the dependency between jump and ammunition temperature, seeing which zeroing method has a higher hit probability when the temperature at the time of the individual zeroing exercise differs from the temperature at the time of combat is of interest. To do this, we can plot the difference in hit probability, $\Delta P_{Hit} = P(Hit_{IZ}) - P(Hit_{FZ})$, as a function of both of these temperatures for assumed values of the variance components and the temperature-dependent slope. Estimates of these parameters are available for most ammunition in the U.S. Army arsenal. As an example, consider a hypothetical training round with a temperature-dependent slope of $\beta = 0.004$ mils per degree Fahrenheit.[2*] Assume also that the variance components relative to the length ($L$) of the square target are $\sigma_{R_x} = \sigma_{R_y} = \sigma_{O_x} = \sigma_{O_y} = L/6$ and $\sigma_{T_x} = \sigma_{T_y} = 0.97 \times L/6$. Figure 1 is a 3-D plot generated by MATLAB® of $\Delta P_{Hit}$ as a function of the firing and zeroing temperature for this baseline case. The black lines are contours for $\Delta P_{Hit} = 0$. Yellow and orange regions indicate temperature conditions for which $\Delta P_{Hit} > 0$ (i.e., when individual zero produces the higher hit probability). On the other hand, blue regions indicate when fleet zero gives better hit probability. From the figure, we see that when firing and zeroing temperatures are equal, individual zero is the preferred policy, especially when the temperatures are toward their extremes. Conversely, if the firing and zeroing temperatures differ greatly, the preferred procedure is fleet zero (blue regions). For this baseline case, we also obtain the curious result that tanks which individually zero at 68$^{o}$ F will have the same hit probability as using a fleet zero for *any* firing temperature.

One can next study the effect of increasing (or decreasing) any of the variance components on $\Delta P_{Hit}$, keeping all other parameters constant.[†] First, consider a change in tank-to-tank dispersion. Because $\sigma_T$ is negatively correlated with $P(Hit_{FZ})$ but not correlated at all with $P(Hit_{IZ})$, $\Delta P_{Hit}$ increases with $\sigma_T$. Intuitively, this makes sense – greater tank-to-tank variation increases the number of tanks with large bias and hence the need to individually zero; the opposite holds with less variability. This is confirmed graphically in Figure 2, where relative to the baseline case, a decrease in $\sigma_T$ to a value

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[*] Three of the four ammunition types studied in this report had temperature-dependent slopes ranging from .0041 to .0047 mils per degree Fahrenheit; one ammunition type had no temperature dependency. Therefore, a value of .004 mils is reasonable for the purpose of this example.

[†] For simplification of the ensuing example, we will henceforth assume that the horizontal and vertical variance components are equal and will drop the "X" and "Y" subscripts used previously.

of $0.97*L/8$ makes fleet zero the more favorable policy; the opposite is true if $\sigma_T$ is increased to $0.97*L/4$. In fact, in this latter case, we see that there is a range of temperatures (24° F to 95° F) in which the tank can be individually zeroed and have a better $P(Hit)$ than under fleet zero, no matter what the actual ammunition temperature is at the time of firing. This range of temperatures will hereafter be referred to as the Individual Zero Preferred (IZP) range.

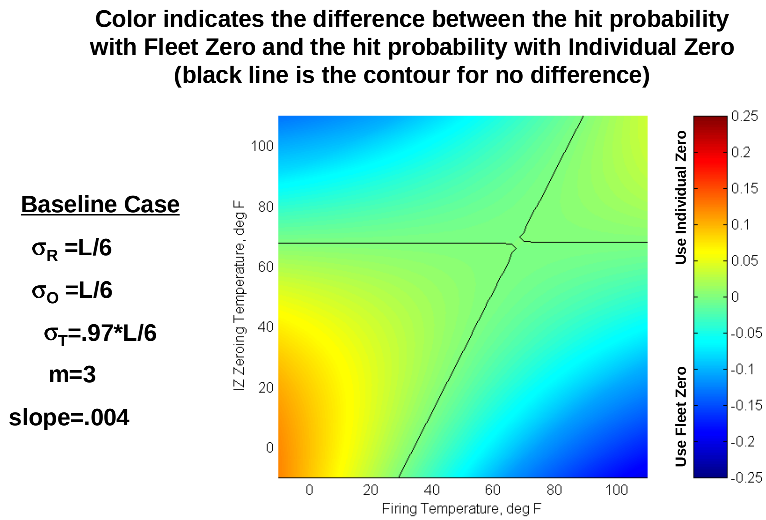


Figure 1. Baseline-case relationship between $\Delta P_{Hit}$ and the ammunition temperatures at the time of firing and zeroing.

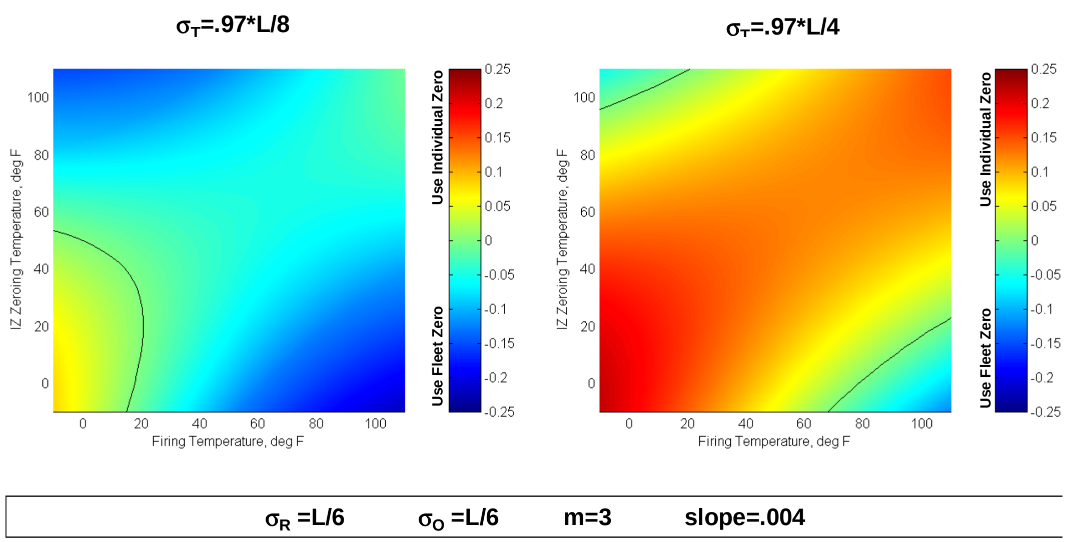


Figure 2 (a) and (b). Effect of changing $\sigma_T$, while holding all other parameters fixed at baseline value.

If occasion-to-occasion dispersion is the free parameter, it is not intuitive which zeroing method becomes more preferable since, for example, both $P(Hit_{FZ})$ and $P(Hit_{IZ})$ decrease when $\sigma_O$ increases. Therefore, one can turn to the 3-D color charts to see which hit probability is influenced the most by a change in $\sigma_O$. Figure 3(a) shows that a reduction in $\sigma_O$ creates an IZP range of approximately 52° F to 77° F. On the other hand, as $\sigma_O$ increases, Figure 3(b) shows fleet zero becoming the more preferable policy. This is easily explained by noting that the expressions for the variance of jump (both in vertical and horizontal) under fleet zero have a $\sigma_O^2$ coefficient of approximately 1.12; in the case of individual zeroing, this same coefficient is larger, namely 2.

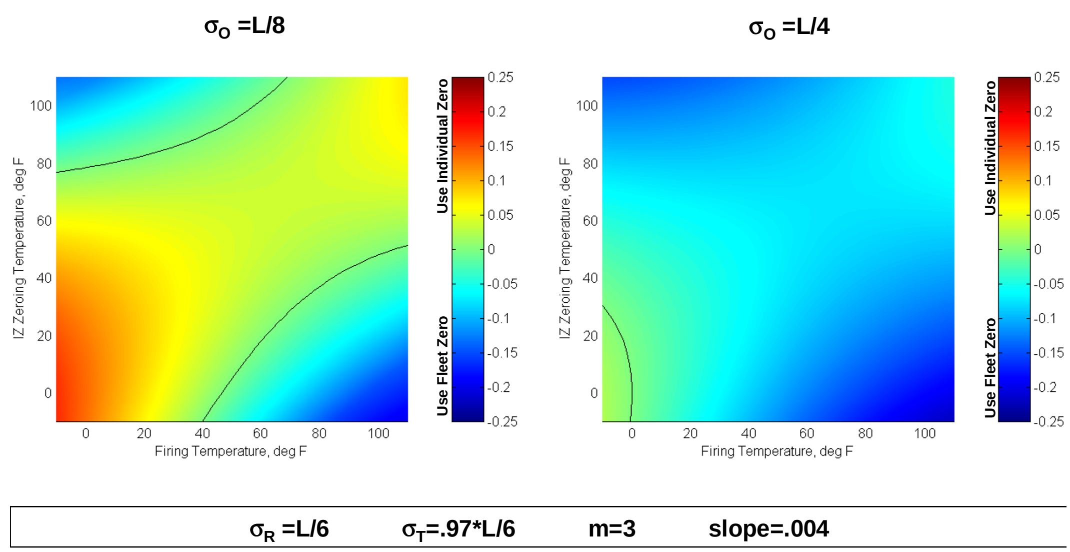


Figure 3 (a) and (b). Effect of changing $\sigma_O$, while holding all other parameters fixed at baseline value.

The effect of a different round-to-round dispersion is similar to that of a change in occasion-to-occasion dispersion, although less pronounced. This is seen in Figure 4(a), where lowering $\sigma_R$ by the same amount as that used for $\sigma_O$ to generate Figure 3(a) yields a smaller IZP range of 62° F to 71° F.

Figure 5 is included to show two extreme cases of operating with a different temperature-dependent slope. Recall the previous discussion (page 9) in which it was shown that under specific conditions (including no temperature dependency), $P(Hit_{FZ}) > P(Hit_{IZ})$ if and only if $\sigma_T < .97\sigma_R$. Figure 5(a) is the trivial extension to this set of conditions, showing that $\sigma_T = .97\sigma_R$ implies $\Delta P_{Hit} = 0$ for all combinations of individual zero and actual firing temperatures. Figure 5(b) is for the case where the temperature dependency is much stronger. Similar to the baseline case, we see that tanks which individually zero at 68° F will be equally as effective as tanks using fleet zero, regardless of the actual firing temperature. The deep orange regions of this figure indicate that there is great potential for improving hit probability if tank commanders are allowed to individually zero at the same extreme temperature as that expected during battle. However, the blue regions indicate the greater risk involved if a tank is individually zeroed at a temperature much different from that encountered during battle.

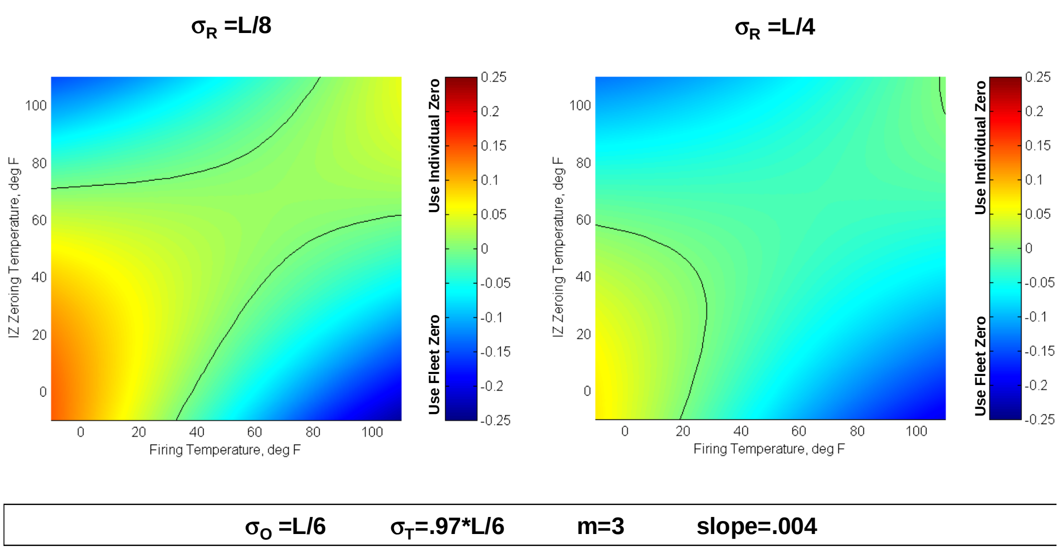


Figure 4 (a) and (b). Effect of changing $\sigma_R$, while holding all other parameters fixed at baseline value.

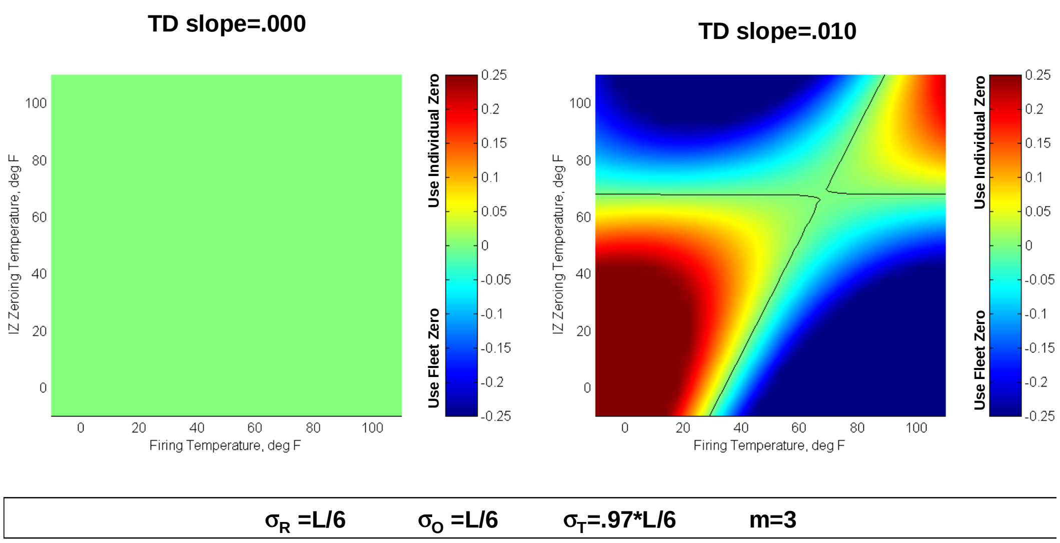


Figure 5 (a) and (b). Effect of changing temperature-dependent slope, while holding all other parameters fixed at baseline value.

**Implications**

The results of the current modeling effort imply some interesting and important information and policy choices. First, we know from previous testing that the magnitude of the various error sources varies by ammunition type. Ammunition type A may have small tank-to-tank errors, while type B may have large tank-to-tank errors. In fact, the trend appears to be that the more energetic the ammunition, the greater the tank-to-tank errors.[3] This means that an optimal zero policy may vary across ammunition types. For example, ammunition types with small tank-to-tank differences could use the fleet zero, and those types that exhibit large tank-to-tank differences could individually zero those ammunition types when needed.

Conversely, a policy that screens tanks could identify those tanks that do not shoot well with the fleet zero and require that they be individually zeroed. This seems to be the policy that is used today during gunnery exercises. Prior to conducting the record tank tables,[*] each tank fires several rounds of ammunition to ensure that the tank is calibrated using the fleet CCF. When a tank cannot demonstrate a calibration in this manner, the tank is zeroed using an individual zero.

Whether a new zeroing policy would return completely to the individual zero method or to some hybrid where only certain ammunition types are individually zeroed across the fleet or where only certain tanks are individually zeroed, the training impacts will be large. The U.S. Army has trained with the fleet zero for nearly two decades and it was successful in Operation Desert Storm. As a result, there is a significant amount of confidence in the policy and a great deal of institutional support. To change the policy requires convincing most of the Armor Force that a significantly better approach exists. In addition, a substantial training effort would be required to prevent the numerous possible errors associated with individually zeroing tanks from manifesting themselves across the fleet because of improper zeroing.

Significantly, the modeling showed that the greater the jump dependency on ammunition temperature, the more sensitive the accuracy of the tank is to the zeroing decision-making process. This means that attention to temperature-related jump effects should continue when developing ammunition. Fortunately, temperature-dependent jump is measurable and consistent across the fleet of tanks.

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[3] Pell, Richard F., "Key to Improve Accuracy: Tighter Gun Tube Specs", Letter to the Editor, *ARMOR*, January-February 1996, pg. 3.

[*] Tank tables are training exercises whereby a tank(s) fire at various targets on a range and are scored on number of hits, time, and procedure. On most of the tank tables, the tanks also maneuver

Therefore, corrections for temperature are possible via the fire control system or temperature conditioning while the ammunition is stored in the tank's bustle.

Finally, the importance of having accurate tanks and the difficulties associated with changing zeroing policies suggest that a root cause analysis to determine why some ammunition types or some tanks require different zero policies is warranted. Certainly, a great deal of understanding and knowledge has been developed over the last few decades, but more work seems necessary if the accuracy of U.S. tanks is to keep pace with the demands for greater hit probability at extended range.

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